

Tables for Group Theory

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This provides the essential tables (character tables, direct products, descent in symmetry and subgroups) required for those using group theory, together with general formulae, examples, and other relevant information.

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Character Tables**Notes:**

(1) Schönflies symbols are given for all point groups. Hermann–Mauguin symbols are given for the 32 crystallographic point groups.

(2) In the groups containing the operation C_5 the following relations are useful:

$$\eta^+ = \frac{1}{2}(1 + 5^{\frac{1}{2}}) = 1.61803\dots = -2 \cos 144^\circ$$

$$\eta^- = \frac{1}{2}(1 - 5^{\frac{1}{2}}) = -0.61803\dots = -2 \cos 72^\circ$$

$$\eta^+ \eta^+ = 1 + \eta^+ \quad \eta^- \eta^- = 1 + \eta^- \quad \eta^+ \eta^- = -1$$

$$\eta^+ + \eta^- = 1 \quad 2 \cos 72^\circ + 2 \cos 144^\circ = -1$$

1. The Groups C_1 , C_s , C_i

C_1 (1)	E
A	1

$C_s=C_h$ (m)	E	σ_h		
A'	1	1	x, y, R_z	x^2, y^2, z^2, xy
A''	1	-1	z, R_x, R_y	yz, xz

$C_i = S_2$ (1)	E	i		
A_g	1	1	R_x, R_y, R_z	$x^2, y^2, z^2,$ xy, xz, yz
A_u	1	-1	x, y, z	

2. The Groups C_n ($n = 2, 3, \dots, 8$)

C_2 (2)	E	C_2					
A	1	1	z, R_z	x^2, y^2, z^2, xy			
B	1	-1	x, y, R_x, R_y	yz, xz			
C_3 (3)	E	C_3	C_3^2			$\varepsilon = \exp(2\pi i/3)$	
A	1	1	1	z, R_z	$x^2 + y^2, z^2$		
E	$\begin{Bmatrix} 1 & \varepsilon & \varepsilon^* \\ 1 & \varepsilon^* & \varepsilon \end{Bmatrix}$	ε	ε^*	ε	$(x, y)(R_x, R_y)$	$(x^2 - y^2, 2xy)(yz, xz)$	
C_4 (4)	E	C_4	C_2	C_4^3			
A	1	1	1	1	z, R_z	$x^2 + y^2, z^2$	
B	1	-1	1	-1		$x^2 - y^2, 2xy$	
E	$\begin{Bmatrix} 1 & i & -1 & -i \\ 1 & -i & -1 & i \end{Bmatrix}$	i	$-i$	i	$(x, y)(R_x, R_y)$	(yz, xz)	
C_5	E	C_5	C_5^2	C_5^3	C_5^4	$\varepsilon = \exp(2\pi i/5)$	
A	1	1	1	1	1	z, R_z $x^2 + y^2, z^2$	
E_1	$\begin{Bmatrix} 1 & \varepsilon & \varepsilon^2 & \varepsilon^{*2} & \varepsilon^* \\ 1 & \varepsilon^* & \varepsilon^{*2} & \varepsilon^2 & \varepsilon \end{Bmatrix}$	ε	ε^2	ε^{*2}	ε^*	$(x, y)(R_x, R_y)$ (yz, xz)	
E_2	$\begin{Bmatrix} 1 & \varepsilon^2 & \varepsilon^* & \varepsilon & \varepsilon^{*2} \\ 1 & \varepsilon^{*2} & \varepsilon & \varepsilon^* & \varepsilon^2 \end{Bmatrix}$	ε^2	ε^*	ε	ε^{*2}	$(x^2 - y^2, 2xy)$	
C_6 (6)	E	C_6	C_3	C_2	C_3^2	C_6^5	$\varepsilon = \exp(2\pi i/6)$
A	1	1	1	1	1	1	z, R_z $x^2 + y^2, z^2$
B	1	-1	1	-1	1	-1	
E_1	$\begin{Bmatrix} 1 & \varepsilon & -\varepsilon^* & -1 & -\varepsilon & \varepsilon^* \\ 1 & \varepsilon^* & -\varepsilon & -1 & -\varepsilon^* & \varepsilon \end{Bmatrix}$	ε	$-\varepsilon^*$	-1	$-\varepsilon$	ε^*	(x, y) (R_x, R_y) (xy, yz)
E_2	$\begin{Bmatrix} 1 & -\varepsilon^* & -\varepsilon & 1 & -\varepsilon^* & -\varepsilon \\ 1 & -\varepsilon & -\varepsilon^* & 1 & -\varepsilon & -\varepsilon^* \end{Bmatrix}$	$-\varepsilon^*$	$-\varepsilon$	1	$-\varepsilon^*$	$-\varepsilon$	$(x^2 - y^2, 2xy)$

2. The Groups C_n ($n = 2, 3, \dots, 8$) (cont..)

C_7	E	C_7	C_7^2	C_7^3	C_7^4	C_7^5	C_7^6		$\varepsilon = \exp(2\pi i/7)$
A	1	1	1	1	1	1	1	z, R_z	$x^2 + y^2, z^2$
E_1	$\begin{Bmatrix} 1 & \varepsilon & \varepsilon^2 & \varepsilon^3 & \varepsilon^{*3} & \varepsilon^{*2} & \varepsilon^* \\ 1 & \varepsilon^* & \varepsilon^{*2} & \varepsilon^{*3} & \varepsilon^3 & \varepsilon^2 & \varepsilon \end{Bmatrix}$							(x, y) (R_x, R_y)	(xz, yz)
E_2	$\begin{Bmatrix} 1 & \varepsilon^2 & \varepsilon^{*3} & \varepsilon^* & \varepsilon & \varepsilon^3 & \varepsilon^{*2} \\ 1 & \varepsilon^{*2} & \varepsilon^3 & \varepsilon & \varepsilon^* & \varepsilon^{*3} & \varepsilon^2 \end{Bmatrix}$								$(x^2 - y^2, 2xy)$
E_3	$\begin{Bmatrix} 1 & \varepsilon^3 & \varepsilon^* & \varepsilon^2 & \varepsilon^{*2} & \varepsilon & \varepsilon^{*3} \\ 1 & \varepsilon^{*3} & \varepsilon & \varepsilon^{*2} & \varepsilon^2 & \varepsilon^* & \varepsilon^3 \end{Bmatrix}$								

C_8	E	C_8	C_4	C_2	C_4^3	C_8^3	C_8^5	C_8^7		$\varepsilon = \exp(2\pi i/8)$
A	1	1	1	1	1	1	1	1	z, R_z	$x^2 + y^2, z^2$
B	1	-1	1	1	1	-1	-1	-1		
E_1	$\begin{Bmatrix} 1 & \varepsilon & i & -1 & -i & -\varepsilon^* & -\varepsilon & \varepsilon^* \\ 1 & \varepsilon^* & -i & -1 & i & -\varepsilon & -\varepsilon^* & \varepsilon \end{Bmatrix}$								(x, y) (R_x, R_y)	(xz, yz)
E_2	$\begin{Bmatrix} 1 & i & -1 & 1 & -1 & -i & i & -i \\ 1 & -i & -1 & 1 & -1 & i & -i & i \end{Bmatrix}$									$(x^2 - y^2, 2xy)$
E_3	$\begin{Bmatrix} 1 & -\varepsilon & i & -1 & -i & \varepsilon^* & \varepsilon & -\varepsilon^* \\ 1 & -\varepsilon^* & -i & -1 & i & \varepsilon & \varepsilon^* & -\varepsilon \end{Bmatrix}$									

3. The Groups D_n ($n = 2, 3, 4, 5, 6$)

D_2 (222)	E	$C_2(z)$	$C_2(y)$	$C_2(x)$		
A	1	1	1	1		x^2, y^2, z^2
B ₁	1	1	-1	-1	z, R_z	xy
B ₂	1	-1	1	-1	y, R_y	xz
B ₃	1	-1	-1	1	x, R_x	yz

D_3 (32)	E	$2C_3$	$3C_2$			
A ₁	1	1	1			$x^2 + y^2, z^2$
A ₂	1	1	-1	z, R_z		
E	2	-1	0	$(x, y)(R_x, R_y)$	$(x^2 - y^2, 2xy)$	(xz, yz)

D_4 (422)	E	$2C_4$	$C_2(=C_4^2)$	$2C_2'$	$2C_2''$		
A ₁	1	1	1	1	1		$x^2 + y^2, z^2$
A ₂	1	1	1	-1	-1	z, R_z	
B ₁	1	-1	1	1	-1		$x^2 - y^2$
B ₂	1	-1	1	-1	1		xy
E	2	0	-2	0	0	$(x, y)(R_x, R_y)$	(xz, yz)

D_5	E	$2C_5$	$2C_5^2$	$5C_2$		
A ₁	1	1	1	1		$x^2 + y^2, z^2$
A ₂	1	1	1	-1	z, R_z	
E ₁	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	$(x, y)(R_x, R_y)$	(xz, yz)
E ₂	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0		$(x^2 - y^2, 2xy)$

D_6 (622)	E	$2C_6$	$2C_3$	C_2	$3C_2'$	$3C_2''$		
A ₁	1	1	1	1	1	1		$x^2 + y^2, z^2$
A ₂	1	1	1	1	-1	-1	z, R_z	
B ₁	1	-1	1	-1	1	-1		
B ₂	1	-1	1	-1	-1	1		
E ₁	2	1	-1	-2	0	0	$(x, y)(R_x, R_y)$	(xz, yz)
E ₂	2	-1	-1	2	0	0		$(x^2 - y^2, 2xy)$

4. The Groups C_{nv} ($n = 2, 3, 4, 5, 6$)

C_{2v} ($2mm$)	E	C_2	$\sigma_v(xz)$	$\sigma'_v(yz)$		
A_1	1	1	1	1	z	x^2, y^2, z^2
A_2	1	1	-1	-1	R_z	xy
B_1	1	-1	1	-1	x, R_y	xz
B_2	1	-1	-1	1	y, R_x	yz

C_{3v} ($3m$)	E	$2C_3$	$3\sigma_v$			
A_1	1	1	1	z		$x^2 + y^2, z^2$
A_2	1	1	-1	R_z		
E	2	-1	0	$(x, y)(R_x, R_y)$		$(x^2 - y^2, 2xy)(xz, yz)$

C_{4v} ($4mm$)	E	$2C_4$	C_2	$2\sigma_v$	$2\sigma_d$		
A_1	1	1	1	1	1	z	$x^2 + y^2, z^2$
A_2	1	1	1	-1	-1	R_z	
B_1	1	-1	1	1	-1		$x^2 - y^2$
B_2	1	-1	1	-1	1		xy
E	2	0	-2	0	0	$(x, y)(R_x, R_y)$	(xz, yz)

C_{5v}	E	$2C_5$	$2C_5^2$	$5\sigma_v$		
A_1	1	1	1	1	z	$x^2 + y^2, z^2$
A_2	1	1	1	-1	R_z	
E_1	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	$(x, y)(R_x, R_y)$	(xz, yz)
E_2	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0		$(x^2 - y^2, 2xy)$

C_{6v} ($6mm$)	E	$2C_6$	$2C_3$	C_2	$3\sigma_v$	$3\sigma_d$		
A_1	1	1	1	1	1	1	z	$x^2 + y^2, z^2$
A_2	1	1	1	1	-1	-1	R_z	
B_1	1	-1	1	-1	1	-1		
B_2	1	-1	1	-1	-1	1		
E_1	2	1	-1	-2	0	0	$(x, y)(R_x, R_y)$	(xz, yz)
E_2	2	-1	-1	2	0	0		$(x^2 - y^2, 2xy)$

5. The Groups C_{nh} ($n = 2, 3, 4, 5, 6$)

C_{2h} ($2/m$)	E	C_2	I	σ_h		
A_g	1	1	1	1	R_z	x^2, y^2, z^2, xy
B_g	1	-1	1	-1	R_x, R_y	xz, yz
A_u	1	1	-1	-1	z	
B_u	1	-1	-1	1	x, y	

C_{3h} ($\bar{6}$)	E	C_3	C_3^2	σ_h	S_3	S_3^5		$\varepsilon = \exp(2\pi i/3)$
A'	1	1	1	1	1	1	R_z	$x^2 + y^2, z^2$
E'	$\left\{ \begin{array}{cccccc} 1 & \varepsilon & \varepsilon^* & 1 & \varepsilon & \varepsilon^* \\ 1 & \varepsilon^* & \varepsilon & 1 & \varepsilon^* & \varepsilon \end{array} \right\}$						(x, y)	$(x^2 - y^2, 2xy)$
A''	1	1	1	-1	-1	-1	z	
E''	$\left\{ \begin{array}{cccccc} 1 & \varepsilon & \varepsilon^* & -1 & -\varepsilon & -\varepsilon^* \\ 1 & \varepsilon^* & \varepsilon & -1 & -\varepsilon^* & -\varepsilon \end{array} \right\}$						(R_x, R_y)	(xz, yz)

C_{4h} ($4/m$)	E	C_4	C_2	C_4^3	i	S_4^3	σ_h	S_4		
A_g	1	1	1	1	1	1	1	1	R_z	$x^2 + y^2, z^2$
B_g	1	-1	1	-1	1	-1	1	-1		$(x^2 - y^2, 2xy)$
E_g	$\left\{ \begin{array}{cccccc} 1 & i & -1 & -i & 1 & i & -1 & -i \\ 1 & -i & -1 & i & 1 & -i & -1 & i \end{array} \right\}$								(R_x, R_y)	(xz, yz)
A_u	1	1	1	1	-1	-1	-1	-1	z	
B_u	1	-1	1	-1	-1	1	-1	1		
E_u	$\left\{ \begin{array}{cccccc} 1 & i & -1 & -i & -1 & -i & 1 & i \\ 1 & -i & -1 & i & -1 & i & 1 & -i \end{array} \right\}$								(x, y)	

6. The Groups D_{nh} ($n = 2, 3, 4, 5, 6$)

D_{2h} (mmm)	E	$C_2(z)$	$C_2(y)$	$C_2(x)$	i	$\sigma(xy)$	$\sigma(xz)$	$\sigma(yz)$	
A_g	1	1	1	1	1	1	1	1	x^2, y^2, z^2
B_{1g}	1	1	-1	-1	1	1	-1	-1	R_z xy
B_{2g}	1	-1	1	-1	1	-1	1	-1	R_y xz
B_{3g}	1	-1	-1	1	1	-1	-1	1	R_x yz
A_u	1	1	1	1	-1	-1	-1	-1	
B_{1u}	1	1	-1	-1	-1	-1	1	1	z
B_{2u}	1	-1	1	-1	-1	1	-1	1	y
B_{3u}	1	-1	-1	1	-1	1	1	-1	x

D_{3h} ($\bar{6}$) $m2$	E	$2C_3$	$3C_2$	σ_h	$2S_3$	$3\sigma_v$	
A'_1	1	1	1	1	1	1	$x^2 + y^2, z^2$
A'_2	1	1	-1	1	1	-1	R_z
E'	2	-1	0	2	-1	0	(x, y) $(x^2 - y^2, 2xy)$
A''_1	1	1	1	-1	-1	-1	
A''_2	1	1	-1	-1	-1	1	z
E''	2	-1	0	-2	1	0	(R_x, R_y) (xy, yz)

D_{4h} ($4/mmm$)	E	$2C_4$	C_2	$2C'_2$	$2C''_2$	i	$2S_4$	σ_h	$2\sigma_v$	$2\sigma_d$	
A_{1g}	1	1	1	1	1	1	1	1	1	1	$x^2 + y^2, z^2$
A_{2g}	1	1	1	-1	-1	1	1	1	-1	-1	R_z
B_{1g}	1	-1	1	1	-1	1	-1	1	1	-1	$x^2 - y^2$
B_{2g}	1	-1	1	-1	1	1	-1	1	-1	1	xy
E_g	2	0	-2	0	0	2	0	-2	0	0	(R_x, R_y) (xz, yz)
A_{1u}	1	1	1	1	1	-1	-1	-1	-1	-1	
A_{2u}	1	1	1	-1	-1	-1	-1	-1	1	1	z
B_{1u}	1	-1	1	1	-1	-1	1	-1	-1	1	
B_{2u}	1	-1	1	-1	1	-1	1	-1	1	-1	
E_u	2	0	-2	0	0	-2	0	2	0	0	(x, y)

6. The Groups D_{nh} ($n = 2, 3, 4, 5, 6$) (cont...)

D_{5h}	E	$2C_5$	$2C_5^2$	$5C_2$	σ_h	$2S_5$	$2S_5^3$	$5\sigma_v$	
A_1'	1	1	1	1	1	1	1	1	$x^2 + y^2, z^2$
A_2'	1	1	1	-1	1	1	1	-1	R_z
E_1'	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	(x, y)
E_2'	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	$(x^2 - y^2, 2xy)$
A_1''	1	1	1	1	-1	-1	-1	-1	
A_2''	1	1	1	-1	-1	-1	-1	1	z
E_1''	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	-2	$-2 \cos 72^\circ$	$-2 \cos 144^\circ$	0	(R_x, R_y) (xy, yz)
E_2''	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	-2	$-2 \cos 144^\circ$	$-2 \cos 72^\circ$	0	

D_{6h} ($6/mmm$)	E	$2C_6$	$2C_3$	C_2	$3C_2'$	$3C_2''$	i	$2S_3$	$2S_6$	σ_h	$3\sigma_d$	$3\sigma_v$	
A_{1g}	1	1	1	1	1	1	1	1	1	1	1	1	$x^2 + y^2, z^2$
A_{2g}	1	1	1	1	-1	-1	1	1	1	1	-1	-1	R_z
B_{1g}	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	
B_{2g}	1	-1	1	-1	-1	1	1	-1	1	-1	-1	1	
E_{1g}	2	1	-1	-2	0	0	2	1	-1	-2	0	0	$(R_x - R_y)$ (xz, yz)
E_{2g}	2	-1	-1	2	0	0	2	-1	-1	2	0	0	$(x^2 - y^2, 2xy)$
A_{1u}	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	
A_{2u}	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	z
B_{1u}	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	
B_{2u}	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1	
E_{1u}	2	1	-1	-2	0	0	-2	-1	1	2	0	0	(x, y)
E_{2u}	2	-1	-1	2	0	0	-2	1	1	-2	0	0	

7. The Groups D_{nd} ($n = 2, 3, 4, 5, 6$)

$D_{2d} = V_d$ ($\bar{4}2m$)	E	$2S_4$	C_2	$2C'_2$	$2\sigma_d$		
A_1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	1	-1	-1	R_z	
B_1	1	-1	1	1	-1		$x^2 - y^2$
B_2	1	-1	1	-1	1	z	xy
E	2	0	-2	0	0	(x, y) (R_x, R_y)	(xz, yz)

D_{3d} ($\bar{3}m$)	E	$2C_3$	$3C_2$	i	$2S_6$	$3\sigma_d$	
A_{1g}	1	1	1	1	1	1	$x^2 + y^2, z^2$
A_{2g}	1	1	-1	1	1	-1	R_z
E_g	2	-1	0	2	-1	0	(R_x, R_y) $(x^2 - y^2, 2xy)$ (xz, yz)
A_{1u}	1	1	1	-1	-1	-1	
A_{2u}	1	1	-1	-1	-1	1	z
E_u	2	-1	0	-2	1	0	(x, y)

D_{4d}	E	$2S_8$	$2C_4$	$2S_8^3$	C_2	$4C'_2$	$4\sigma_d$	
A_1	1	1	1	1	1	1	1	$x^2 + y^2, z^2$
A_2	1	1	1	1	1	-1	-1	R_z
B_1	1	-1	1	-1	1	1	-1	
B_2	1	-1	1	-1	1	-1	1	z
E_1	2	$\sqrt{2}$	0	$-\sqrt{2}$	-2	0	0	(x, y)
E_2	2	0	-2	0	2	0	0	$(x^2 - y^2, 2xy)$
E_3	2	$-\sqrt{2}$	0	$\sqrt{2}$	-2	0	0	(R_x, R_y) (xz, yz)

7. The Groups D_{nd} ($n = 2, 3, 4, 5, 6$) (cont..)

D_{5d}	E	$2C_5$	$2C_5^2$	$5C_2$	i	$2S_{10}^3$	$2S_{10}$	$5\sigma_d$	
A_{1g}	1	1	1	1	1	1	1	1	$x^2 + y^2, z^2$
A_{2g}	1	1	1	-1	1	1	1	-1	R_z
E_{1g}	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	(R_x, R_y)
E_{2g}	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	$(x^2 - y^2, 2xy)$
A_{1u}	1	1	1	1	-1	-1	-1	-1	
A_{2u}	1	1	1	-1	-1	-1	-1	1	z
E_{1u}	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	-2	$-2 \cos 72^\circ$	$-2 \cos 144^\circ$	0	(x, y)
E_{2u}	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	-2	$-2 \cos 144^\circ$	$-2 \cos 72^\circ$	0	

D_{6d}	E	$2S_{12}$	$2C_6$	$2S_4$	$2C_3$	$2S_{12}^5$	C_2	$6C'_2$	$6\sigma_d$	
A_1	1	1	1	1	1	1	1	1	1	$x^2 + y^2, z^2$
A_2	1	1	1	1	1	1	1	-1	-1	R_z
B_1	1	-1	1	-1	1	-1	1	1	-1	
B_2	1	-1	1	-1	1	-1	1	-1	1	z
E_1	2	$\sqrt{3}$	1	0	-1	$-\sqrt{3}$	-2	0	0	(x, y)
E_2	2	1	-1	-2	-1	1	2	0	0	$(x^2 - y^2, 2xy)$
E_3	2	0	-2	0	2	0	-2	0	0	
E_4	2	-1	-1	2	-1	-1	2	0	0	
E_5	2	$-\sqrt{3}$	1	0	-1	$\sqrt{3}$	-2	0	0	(R_x, R_y) (xy, yz)

8. The Groups S_n ($n = 4, 6, 8$)

S_4 ($\bar{4}$)	E	S_4	C_2	S_4^3		
A	1	1	1	1	R_z	$x^2 + y^2, z^2$
B	1	-1	1	-1	z	$(x^2 - y^2, 2xy)$
E	$\left\{ \begin{array}{cccc} 1 & i & -1 & -i \\ 1 & -i & -1 & i \end{array} \right\}$				(x, y) (R_x, R_y)	(xz, yz)

S_6 ($\bar{3}$)	E	C_3	C_3^2	i	S_6^5	S_6		$\varepsilon = \exp(2\pi i/3)$
A_g	1	1	1	1	1	1	R_z	$x^2 + y^2, z^2$
E_g	$\left\{ \begin{array}{cccccc} 1 & \varepsilon & \varepsilon^* & 1 & \varepsilon & \varepsilon^* \\ 1 & \varepsilon^* & \varepsilon & 1 & \varepsilon^* & \varepsilon \end{array} \right\}$						(R_x, R_y)	$(x^2 - y^2, 2xy)$ (xy, yz)
A_u	1	1	1	-1	-1	-1	z	
E_u	$\left\{ \begin{array}{cccccc} 1 & \varepsilon & \varepsilon^* & 1 & -\varepsilon & -\varepsilon^* \\ 1 & \varepsilon^* & \varepsilon & 1 & -\varepsilon^* & -\varepsilon \end{array} \right\}$						(x, y)	

S_8	E	S_8	C_4	S_8^3	C_2	S_8^5	C_4^3	S_8^7		$\varepsilon = \exp(2\pi i/8)$
A	1	1	1	1	1	1	1	1	R_z	$x^2 + y^2, z^2$
B	1	-1	1	-1	1	-1	1	-1	z	
E_1	$\left\{ \begin{array}{cccccc} 1 & \varepsilon & i & -\varepsilon^* & -1 & -\varepsilon & -i & \varepsilon^* \\ 1 & \varepsilon^* & -i & -\varepsilon & -1 & -\varepsilon^* & i & \varepsilon \end{array} \right\}$								(x, y)	
E_2	$\left\{ \begin{array}{cccccc} 1 & i & -1 & -i & 1 & i & -1 & -i \\ 1 & -i & -1 & i & 1 & -i & -1 & i \end{array} \right\}$									$(x^2 - y^2, 2xy)$
E_3	$\left\{ \begin{array}{cccccc} 1 & -\varepsilon^* & -i & \varepsilon & -1 & \varepsilon^* & i & -\varepsilon \\ 1 & -\varepsilon & i & \varepsilon^* & -1 & \varepsilon & -i & -\varepsilon^* \end{array} \right\}$								(R_x, R_y)	(xy, yz)

9. The Cubic Groups

T (23)	E	$4C_3$	$4C_3^2$	$3C_2$		$\varepsilon = \exp(2\pi i/3)$
A	1	1	1	1		$x^2 + y^2 + z^2$
E	$\begin{Bmatrix} 1 & \varepsilon & \varepsilon^* & 1 \\ 1 & \varepsilon^* & \varepsilon & 1 \end{Bmatrix}$					$(\sqrt{3}(x^2 - y^2)2z^2 - x^2 - y^2)$
T	3	0	0	-1	(x, y, z) (R_x, R_y, R_z)	(xy, xz, yz)

T_d ($\bar{4}3m$)	E	$8C_3$	$3C_2$	$6S_4$	$6\sigma_d$	
A ₁	1	1	1	1	1	$x^2 + y^2 + z^2$
A ₂	1	1	1	-1	-1	
E	2	-1	2	0	0	$(2z^2 - x^2 - y^2, \sqrt{3}(x^2 - y^2))$
T ₁	3	0	-1	1	-1	(R_x, R_y, R_z)
T ₂	3	0	-1	-1	1	(x, y, z) (xy, xz, yz)

T_h ($m\bar{3}$)	E	$4C_3$	$4C_3^2$	$3C_2$	i	$4S_6$	$4S_6^5$	$3\sigma_d$	$\varepsilon = \exp(2\pi i/3)$
A _g	1	1	1	1	1	1	1	1	$x^2 + y^2 + z^2$
E _g	$\begin{Bmatrix} 1 & \varepsilon & \varepsilon^* & 1 & 1 & \varepsilon & \varepsilon^* & 1 \\ 1 & \varepsilon^* & \varepsilon & 1 & 1 & \varepsilon^* & \varepsilon & 1 \end{Bmatrix}$								$(2z^2 - x^2 - y^2, \sqrt{3}(x^2 - y^2))$
T _g	3	0	0	-1	3	0	0	-1	(R_x, R_y, R_z) (xy, yz, xz)
A _u	1	1	1	1	-1	-1	-1	-1	
E _u	$\begin{Bmatrix} 1 & \varepsilon & \varepsilon^* & 1 & -1 & -\varepsilon & -\varepsilon^* & -1 \\ 1 & \varepsilon^* & \varepsilon & 1 & -1 & -\varepsilon^* & -\varepsilon & -1 \end{Bmatrix}$								
T _u	3	0	0	-1	-3	0	0	1	(x, y, z)

O (432)	E	$8C_3$	$3C_2$	$6C_4$	$6C_2'$	
A ₁	1	1	1	1	1	$x^2 + y^2 + z^2$
A ₂	1	1	1	-1	-1	
E	2	-1	2	0	0	$(2z^2 - x^2 - y^2, \sqrt{3}(x^2 - y^2))$
T ₁	3	0	-1	1	-1	(x, y, z) (R_x, R_y, R_z)
T ₂	3	0	-1	-1	1	(xy, xz, yz)

9. The Cubic Groups (cont...)

O_h ($m\bar{3}m$)	E	$8C_3$	$6C_2$	$6C_4$	$3C_2$ ($= C_4^2$)	i	$6S_4$	$8S_6$	$3\sigma_h$	$6\sigma_d$	
A_{1g}	1	1	1	1	1	1	1	1	1	1	$x^2 + y^2 + z^2$
A_{2g}	1	1	-1	-1	1	1	-1	1	1	-1	
E_g	2	-1	0	0	2	2	0	-1	2	0	$(2z^2 - x^2 - y^2,$ $\sqrt{3}(x^2 - y^2))$
T_{1g}	3	0	-1	1	-1	3	1	0	-1	-1	(R_x, R_y, R_z)
T_{2g}	3	0	1	-1	-1	3	-1	0	-1	1	(xy, xz, yz)
A_{1u}	1	1	1	1	1	-1	-1	-1	-1	-1	
A_{2u}	1	1	-1	-1	1	-1	1	-1	-1	1	
E_u	2	-1	0	0	2	-2	0	1	-2	0	
T_{1u}	3	0	-1	1	-1	-3	-1	0	1	1	(x, y, z)
T_{2u}	3	0	1	-1	-1	-3	1	0	1	-1	

10. The Groups I, I_h

I	E	$12C_5$	$12C_5^2$	$20C_3$	$15C_2$		$\eta^\pm = \frac{1}{2} \left(1 \pm 5^{\frac{1}{2}} \right)$
A	1	1	1	1	1		$x^2 + y^2 + z^2$
T_1	3	η^+	η^-	0	-1	(x, y, z) (R_x, R_y, R_z)	
T_2	3	η^-	η^+	0	-1		
G	4	-1	-1	1	0		
H	5	0	0	-1	1		$(2z^2 - x^2 - y^2,$ $\sqrt{3}(x^2 - y^2)$ $xy, yz, zx)$

I_h	E	$12C_5$	$12C_5^2$	$20C_3$	$15C_2$	i	$12S_{10}$	$12S_{10}^3$	$20S_6$	15σ		$\eta^\pm = \frac{1}{2} \left(1 \pm 5^{\frac{1}{2}} \right)$
A_g	1	1	1	1	1	1	1	1	1	1		$x^2 + y^2 + z^2$
T_{1g}	3	η^+	η^-	0	-1	3	η^-	η^+	0	-1	(R_x, R_y, R_z)	
T_{2g}	3	η^-	η^+	0	-1	3	η^+	η^-	0	-1		
G_g	4	-1	-1	1	0	4	-1	-1	1	0		
H_g	5	0	0	-1	1	5	0	0	-1	1		$(2z^2 - x^2 - y^2,$ $\sqrt{3}(x^2 - y^2))$ (xy, yz, zx)
A_u	1	1	1	1	1	-1	-1	-1	-1	-1		
T_{1u}	3	η^+	η^-	0	-1	-3	η^-	η^+	0	1	(x, y, z)	
T_{2u}	3	η^-	η^+	0	-1	-3	η^+	η^-	0	1		
G_u	4	-1	-1	1	0	-4	1	1	-1	0		
H_u	5	0	0	-1	1	-5	0	0	1	-1		

11. The Groups $C_{\infty v}$ and $D_{\infty h}$

$C_{\infty v}$	E	$2C_{\infty}^{\phi}$...	$\infty\sigma_v$		
$A_1 \equiv \Sigma^+$	1	1	...	1	z	$x^2 + y^2, z^2$
$A_2 \equiv \Sigma^-$	1	1	...	-1	R_z	
$E_1 \equiv \Pi$	2	$2 \cos \phi$...	0	$(x, y) (R_x, R_y)$	(xz, yz)
$E_2 \equiv \Delta$	2	$2 \cos 2\phi$...	0		$(x^2 - y^2, 2xy)$
$E_3 \equiv \Phi$	2	$2 \cos 3\phi$...	0		
...		
...		

$D_{\infty h}$	E	$2C_{\infty}^{\phi}$...	$\infty\sigma_v$	i	$2S_{\infty}^{\phi}$...	∞C_2	
Σ_g^+	1	1	...	1	1	1	...	1	$x^2 + y^2, z^2$
Σ_g^-	1	1	...	-1	1	1	...	-1	R_z
Π_g	2	$2 \cos \phi$...	0	2	$-2 \cos \phi$...	0	$(R_x, R_y) (xz, yz)$
Δ_g	2	$2 \cos 2\phi$...	0	2	$2 \cos 2\phi$...	0	$(x^2 - y^2, 2xy)$
...	
Σ_u^+	1	1	...	1	-1	-1	...	-1	z
Σ_u^-	1	1	...	-1	-1	-1	...	1	
Π_u	2	$2 \cos \phi$...	0	-2	$2 \cos \phi$...	0	(x, y)
Δ_u	2	$2 \cos 2\phi$...	0	-2	$-2 \cos 2\phi$...	0	
...	

12. The Full Rotation Group (SU₂ and R₃)

$$\chi^{(j)}(\phi) = \begin{cases} \frac{\sin\left(j + \frac{1}{2}\right)\phi}{\sin\frac{1}{2}\phi} & \phi \neq 0 \\ 2j+1 & \phi = 0 \end{cases}$$

Notation : Representation labelled $\Gamma^{(j)}$ with $j = 0, 1/2, 1, 3/2, \dots, \infty$, for R_3 j is confined to integral values (and written l or L) and the labels $S \equiv \Gamma^{(0)}$, $P \equiv \Gamma^{(1)}$, $D \equiv \Gamma^{(2)}$, etc. are used.

Direct Products

1. General rules

(a) For point groups in the lists below that have representations A, B, E, T without subscripts, read $A_1 = A_2 = A$, etc.

(b)

	g	u		'	"
g	g	u		'	"
u		g		"	'

(c) Square brackets [] are used to indicate the representation spanned by the antisymmetrized product of a degenerate representation with itself.

Examples

For D_{3h} $E' \times E'' = A_1'' + A_2'' + E$

For D_{6h} $E_{1g} \times E_{2g} = 2B_g + E_{1g}$.

2. For $C_2, C_3, C_6, D_3, D_6, C_{2v}, C_{3v}, C_{6v}, C_{2h}, C_{3h}, C_{6h}, D_{3h}, D_{6h}, D_{3d}, S_6$

	A_1	A_2	B_1	B_2	E_1	E_2
A_1	A_1	A_2	B_1	B_2	E_1	E_2
A_2		A_1	B_2	B_1	E_1	E_2
B_1			A_1	A_2	E_2	E_1
B_2				A_1	E_2	E_1
E_1					$A_1 + [A_2] + E_2$	$B_1 + B_2 + E_1$
E_2						$A_1 + [A_2] + E_2$

3. For D_2, D_{2h}

	A	B_1	B_2	B_3
A	A	B_1	B_2	B_3
B_1		A	B_3	B_2
B_2			A	B_1
B_3				A

4. For $C_4, D_4, C_{4v}, C_{4h}, D_{4h}, D_{2d}, S_4$

	A_1	A_2	B_1	B_2	E
A_1	A_1	A_2	B_1	B_2	E
A_2		A_1	B_2	B_1	E
B_1			A_1	A_2	E
B_2				A_1	E
E					$A_1 + [A_2] + B_1 + B_2$

5. For $C_5, D_5, C_{5v}, C_{5h}, D_{5h}, D_{5d}$

	A_1	A_2	E_1	E_2
A_1	A_1	A_2	E_1	E_2
A_2		A_1	E_1	E_2
E_1			$A_1 + [A_2] + E_2$	$E_1 + E_2$
E_2				$A_1 + [A_2] + E_1$

6. For D_{4d}, S_8

	A_1	A_2	B_1	B_2	E_1	E_2	E_3
A_1	A_1	A_2	B_1	B_2	E_1	E_2	E_3
A_2		A_1	B_2	B_1	E_1	E_2	E_3
B_1			A_1	A_2	E_3	E_2	E_1
B_2				A_1	E_3	E_2	E_1
E_1					$A_1 + [A_2] + E_2$	$E_1 + E_2$	$B_1 + B_2 + E_2$
E_2						$A_1 + [A_2] + B_1 + B_2$	$E_1 + E_3$
E_3							$A_1 + [A_2] + E_2$

7. For T, O, T_h, O_h, T_d

	A_1	A_2	E	T_1	T_2
A_1	A_1	A_2	E	T_1	T_2
A_2		A_1	E	T_2	T_1
E			$A_1 + [A_2] + E$	$T_1 + T_2$	$T_1 + T_2$
T_1				$A_1 + E + [T_1] + T_2$	$A_2 + E + T_1 + T_2$
T_2					$A_1 + E + [T_1] + T_2$

8. For D_{6d}

	A_1	A_2	B_1	B_2	E_1	E_2	E_3	E_4	E_5
A_1	A_1	A_2	B_1	B_2	E_1	E_2	E_3	E_4	E_5
A_2		A_1	B_2	B_1	E_1	E_2	E_3	E_4	E_5
B_1			A_1	A_2	E_5	E_4	E_3	E_2	E_1
B_2				A_1	E_5	E_4	E_3	E_2	E_1
E_1					$A_1 + [A_2] + E_2$	$E_1 + E_3$	$E_2 + E_4$	$E_3 + E_5$	$B_1 + B_2 + E_4$
E_2						$A_1 + [A_2] + E_4$	$E_1 + E_5$	$B_1 + B_2 + E_2$	$E_3 + E_5$
E_3							$A_1 + [A_2] + B_1 + B_2$	$E_1 + E_5$	$E_2 + E_4$
E_4								$A_1 + [A_2] + E_4$	$E_1 + E_3$
E_5									$A_1 + [A_2] + E_2$

9. For I, I_h

	A	T_1	T_2	G	H
A	A	T_1	T_2	G	H
T_1		$A + [T_1] + H$	G + H	$T_2 + G + H$	$T_1 + T_2 + G + H$
T_2			$A + [T_2] + H$	$T_1 + G + H$	$T_1 + T_2 + G + H$
G				$A + [T_1 + T_2] + G + H$	$T_1 + T_2 + G + 2H$
H					$A_1 + [T_1 + T_2 + G] + G + 2H$

10. For $C_{\infty v}, D_{\infty h}$

	Σ^+	Σ^-	Π	Δ
Σ^+	Σ^+	Σ^-	Π	Δ
Σ^-		Σ^+	Π	Δ
Π			$\Sigma^+ + [\Sigma^-] + \Delta$	$\Pi + \Phi$
Δ				$\Sigma^+ + [\Sigma^-] + \Gamma$
:				

Notation

Σ	Π	Δ	Φ	Γ	...
$\Lambda = 0$	1	2	3	4	...

$$\Lambda_1 \times \Lambda_2 = |\Lambda_1 - \Lambda_2| + (\Lambda_1 + \Lambda_2)$$

$$\Lambda \times \Lambda = \Sigma^+ + [\Sigma^-] + (2\Lambda).$$

11. The Full Rotation Group (SU_2 and R_3)

$$\Gamma^{(j)} \times \Gamma^{(j')} = \Gamma^{(j+j')} + \Gamma^{(j+j'-1)} + \dots + \Gamma^{(|j-j'|)}$$

$$\Gamma^{(j)} \times \Gamma^{(j)} = \Gamma^{(2j)} + \Gamma^{(2j-2)} + \dots + \Gamma^{(0)} + [\Gamma^{(2j-1)} + \dots + \Gamma^{(1)}]$$

Extended rotation groups (double groups):
Character tables and direct product tables

D_2^*	E	R	$2C_2(z)$	$2C_2(y)$	$2C_2(x)$
$E_{1/2}$	2	-2	0	0	0

D_3^*	E	R	$2C_3$	$2C_3R$	$3C_2$	$3C_2R$
$E_{1/2}$	2	-2	1	-1	0	0
$E_{3/2}$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right.$	$\left\{ \begin{array}{l} -1 \\ -1 \end{array} \right.$	$\left\{ \begin{array}{l} -1 \\ -1 \end{array} \right.$	$\left\{ \begin{array}{l} 1 \\ 1 \end{array} \right.$	$\left\{ \begin{array}{l} i \\ -i \end{array} \right.$	$\left\{ \begin{array}{l} -i \\ i \end{array} \right.$

D_4	E	R	$2C_4$	$2C_4R$	$2C_2$	$4C_2'$	$4C_2''$
$E_{1/2}$	2	-2	$\sqrt{2}$	$-\sqrt{2}$	0	0	0
$E_{3/2}$	2	-2	$-\sqrt{2}$	$\sqrt{2}$	0	0	0

D_6^*	E	R	$2C_6$	$2C_6R$	$2C_3$	$2C_3R$	$2C_2$	$6C_2'$	$6C_2''$
$E_{1/2}$	2	-2	$\sqrt{3}$	$-\sqrt{3}$	1	-1	0	0	0
$E_{3/2}$	2	-2	$-\sqrt{3}$	$\sqrt{3}$	-1	1	0	0	0
$E_{5/2}$	2	-2	0	0	-2	2	0	0	0

T_d^*	E	R	$8C_3$	$8C_3R$	$6C_2$	$6S_4$	$6S_4R$	$12\sigma_d$
O^*	E	R	$8C_3$	$8C_3R$	$6C_2$	$6C_4$	$6S_4R$	$12C_2'$
$E_{1/2}$	2	-2	1	-1	0	$\sqrt{2}$	$-\sqrt{2}$	0
$E_{5/2}$	2	-2	1	-1	0	$-\sqrt{2}$	$\sqrt{2}$	0
$G_{3/2}$	4	-4	-1	1	0	0	0	0

$$E_{1/2} \times E_{1/2} = [A] + B_1 + B_2 + B_3$$

	$E_{1/2}$	$E_{3/2}$
$E_{1/2}$	$[A_1] + A_2 + E$	2E
$E_{3/2}$		$[A_1] + A_1 + 2A_2$

	$E_{1/2}$	$E_{3/2}$
$E_{1/2}$	$[A_1] + A_2 + E$	$B_1 + B_2 + E$
$E_{3/2}$		$[A_1] + A_2 + E$

	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$
$E_{1/2}$	$[A_1] + A_2 + E_1$	$B_1 + B_2 + E_2$	$E_1 + E_2$
$E_{3/2}$		$[A_1] + A_2 + E_1$	$E_1 + E_2$
$E_{5/2}$			$[A_1] + A_2 + B_1 + B_2$

	$E_{1/2}$	$E_{5/2}$	$E_{3/2}$
$E_{1/2}$	$[A_1] + T_1$	$A_2 + T_2$	$E + T_1 + T_2$
$E_{5/2}$		$[A_1] + T_1$	$E + T_1 + T_2$
$G_{3/2}$			$[A_1 + E + T_2] + A_2 + 2T_1 + T_2$

Direct products of ordinary and extended representations for T_d^ and O^**

	A_1	A_2	E	T_1	T_2
$E_{1/2}$	$E_{1/2}$	$E_{5/2}$	$G_{3/2}$	$E_{1/2} + G_{3/2}$	$E_{5/2} + G_{3/2}$
$E_{5/2}$	$E_{5/2}$	$E_{1/2}$	$G_{3/2}$	$E_{5/2} + G_{3/2}$	$E_{1/2} + G_{3/2}$
$G_{3/2}$	$G_{3/2}$	$G_{3/2}$	$E_{1/2} + E_{5/2} + G_{3/2}$	$E_{1/2} + E_{5/2} + 2G_{3/2}$	$E_{1/2} + E_{5/2} + 2G_{3/2}$

Descent in symmetry and subgroups

The following tables show the correlation between the irreducible representations of a group and those of some of its subgroups. In a number of cases more than one correlation exists between groups. In C_s the σ of the heading indicates which of the planes in the parent group becomes the sole plane of C_s ; in C_{2v} it becomes must be set by a convention); where there are various possibilities for the correlation of C_2 axes and σ planes in D_{4h} and D_{6h} with their subgroups, the column is headed by the symmetry operation of the parent group that is preserved in the descent.

C_{2v}	C_2	C_s $\sigma(zx)$	C_s $\sigma(yz)$
A_1	A	A'	A'
A_2	A	A''	A''
B_1	B	A'	A'
B_2	B	A''	A''

C_{3v}	C_3	C_s
A_1	A	A'
A_2	A	A''
E	E	$A' + A''$

C_{4v}	C_{2v} σ_v	C_{2v} σ_d
A_1	A_1	A_1
A_2	A_2	A_2
B_1	A_1	A_2
B_2	A_2	A_1
E	$B_1 + B_2$	$B_1 + B_2$

[Other subgroups: C_4 , C_2 , C_6]

D_{3h}	C_{3h}	C_{3v}	C_{2v} $\sigma_h \rightarrow \sigma_v$	C_s σ_h	C_s σ_v
A'_1	A'	A_1	A_1	A'	A'
A'_2	A'	A_2	B_2	A'	A''
E'	E'	E	$A_1 + B_2$	$2A'$	$A' + A''$
A''_1	A''	A_2	A_2	A''	A''
A''_2	A''	A_1	B_1	A''	A'
E''	E''	E	$A_2 + B_1$	$2A''$	$A' + A''$

[Other subgroups: D_3 , C_3 , C_2]

Atkins, Child, & Phillips: Tables for Group Theory

D_{4h}	D_{2d}	D_{2d}	D_{2h}	D_{2h}	D_2	D_2	C_{4h}	C_{4v}	C_{2v}	C_{2v}
	$C_2' (\rightarrow C_2')$	$C_2'' (\rightarrow C_2')$	C_2'	C_2''	C_2'	C_2''			C_2, σ_v	C_2, σ_d
A_{1g}	A_1	A_1	A_g	A_g	A	A	A_g	A_1	A_1	A_1
A_{2g}	A_2	A_2	B_{1g}	B_{1g}	B_1	B_1	A_g	A_2	A_2	A_2
B_{1g}	B_1	B_2	A_g	B_{1g}	A	B_1	B_g	B_1	A_1	A_2
B_{2g}	B_2	B_1	B_{1g}	A_g	B_1	A	B_g	B_2	A_2	A_1
E_g	E	E	$B_{2g} + B_{3g}$	$B_{2g} + B_{3g}$	$B_2 + B_3$	$B_2 + B_3$	E_g	E	$B_1 + B_2$	$B_1 + B_2$
A_{1u}	B_1	B_1	A_u	A_u	A	A	A_u	A_2	A_2	A_2
A_{2u}	B_2	B_2	B_{1u}	B_{1u}	B_1	B_1	A_u	A_1	A_1	A_1
B_{1u}	A_1	A_2	A_u	B_{1u}	A	B_1	B_u	B_2	A_2	A_1
B_{2u}	A_2	A_1	B_{1u}	A_u	B_1	A	B_u	B_1	A_1	A_2
E_u	E	E	$B_{2u} + B_{3u}$	$B_{2u} + B_{3u}$	$B_2 + B_3$	$B_2 + B_3$	E_u	E	$B_1 + B_2$	$B_1 + B_2$

Other subgroups: $D_4, C_4, S_4, 3C_{2h}, 3C_s, 3C_2, C_i, (2C_{2v})$

D_6	$D_{3d}C_2''$	$D_{3d}C_2'$	D_{2h}	C_{6v}	C_{3v}	C_{2v}	C_{2v}	C_{2h}	C_{2h}	C_{2h}
			$\sigma_h \rightarrow \sigma(xy)$ $\sigma_v \rightarrow \sigma(yz)$		σ_v	C_2'	C_2''	C_2	C_2'	C_2''
A_{1g}	A_{1g}	A_{1g}	A_g	A_1	A_1	A_1	A_1	A_g	A_g	A_g
A_{2g}	A_{2g}	A_{2g}	B_{1g}	A_2	A_2	B_1	B_1	A_g	B_g	B_g
B_{1g}	A_{2g}	A_{1g}	B_{2g}	B_2	A_2	A_2	B_2	B_g	A_g	B_g
B_{2g}	A_{1g}	A_{2g}	B_{3g}	B_1	A_1	B_2	A_2	B_g	B_g	A_g
E_{1g}	E_g	E_g	$B_{2g} + B_{3g}$	E_1	E	$A_2 + B_2$	$A_2 + B_2$	$2B_g$	$A_g + B_g$	$A_g + B_g$
E_{2g}	E_g	E_g	$A_g + B_{1g}$	E_2	E	$A_1 + B_1$	$A_1 + B_1$	$2A_g$	$A_g + B_g$	$A_g + B_g$
A_{1u}	A_{1u}	A_{1g}	A_u	A_2	A_2	A_2	A_2	A_u	A_u	A_u
A_{2u}	A_{2u}	A_{2g}	B_{1u}	A_1	A_1	B_2	B_2	A_u	B_u	B_u
B_{1u}	A_{2u}	A_{1u}	B_{2u}	B_1	A_1	B_1	B_1	B_u	A_u	B_u
B_{2u}	A_{1u}	A_{2u}	B_{3u}	B_2	A_2	A_1	A_1	B_u	B_u	A_u
E_{1u}	E_u	E_u	$B_{2u} + B_{3u}$	E_1	E	$A_1 + B_1$	$A_1 + B_1$	$2B_u$	$A_u + B_u$	$A_u + B_u$
E_{2u}	E_u	E_u	$A_u + B_{1u}$	E_2	E	$A_2 + B_2$	$A_2 + B_2$	$2A_u$	$A_u + B_u$	$A_u + B_u$

Other subgroups: $D_6, 2D_{3h}, C_{6h}, C_6, C_{3h}, 2D_3, S_6, D_2, C_3, 3C_2, 3C_g, C_i$

T_d	T	D_{2d}	C_{3v}	C_{2v}
A_1	A	A_1	A_1	A_1
A_2	A	B_1	A_2	A_2
E	E	$A_1 + B_1$	E	$A_1 + A_2$
T_1	T	$A_2 + E$	$A_2 + E$	$A_2 + B_1 + B_2$
T_2	T	$B_2 + E$	$A_1 + E$	$A_1 + B_2 + B_1$

Other subgroups: S_4, D_2, C_3, C_2, C_s .

O_h	O	T_d	T_h	D_{4h}	D_{3d}
A_{1g}	A_1	A_1	A_g	A_{1g}	A_{1g}
A_{2g}	A_2	A_2	A_g	B_{1g}	A_{2g}
E_g	E	E	E_g	$A_{1g} + B_{1g}$	E_g
T_{1g}	T_1	T_1	T_g	$A_{2g} + E_g$	$A_{2g} + E_g$
T_{2g}	T_2	T_2	T_g	$B_{2g} + E_g$	$A_{1g} + E_g$
A_{1u}	A_1	A_2	A_u	A_{1u}	A_{1u}
A_{2u}	A_2	A_1	A_u	B_{1u}	B_{1u}
E_u	E	E	E_u	$A_{1u} + B_{1u}$	E_u
T_{1u}	T_1	T_2	T_u	$A_{2u} + E_u$	$A_{2u} + E_u$
T_{2u}	T_2	T_1	T_u	$B_{2u} + E_u$	$A_{1u} + E_u$

Other subgroups: $T, D_4, D_{2d}, C_{4h}, C_{4v}, 2D_{2h}, D_3, C_{3v}, S_6, C_4, S_4, 3C_{2v}, 2D_2, 2C_{2h}, C_3, 2C_2, S_2, C_s$

R_3	O	D_4	D_3
S	A_1	A_1	A_1
P	T_1	$A_2 + E$	$A_2 + E$
D	$E + T_2$	$A_1 + B_1 + B_2 + E$	$A_1 + 2E$
F	$A_2 + T_1 + T_2$	$A_2 + B_1 + B_2 + 2E$	$A_1 + 2A_2 + 2E$
G	$A_1 + E + T_1 + T_2$	$2A_1 + A_2 + B_1 + B_2 + 2E$	$2A_1 + A_2 + 3E$
H	$E + 2T_1 + T_2$	$A_1 + 2A_2 + B_1 + B_2 + 3E$	$A_1 + 2A_2 + 4E$

Notes and Illustrations
General Formulae

(a) Notation

h the *order* (the number of elements) of the group.

$\Gamma^{(i)}$ labels the *irreducible representation*.

$\chi^{(i)}(R)$ the *character* of the operation R in $\Gamma^{(i)}$.

$D_{\mu\nu}^{(i)}(R)$ the $\mu\nu$ element of the *representative matrix* of the operation R in the irreducible representation $\Gamma^{(i)}$.

l_i the *dimension* of $\Gamma^{(i)}$ (the number of rows or columns in the matrices $\mathbf{D}^{(i)}$)

(b) Formulae

(i) *Number of irreducible representations of a group = number of classes.*

$$(ii) \quad \sum_i l_i^2 = h$$

$$(iii) \quad \chi^{(i)}(R) = \sum_{\mu} D_{\mu\mu}^{(i)}(R)$$

(iv) Orthogonality of representations:

$$\sum D_{\mu\nu}^{(i)}(R)^* D_{\mu'\nu'}^{(j)}(R) = (h/l_i) \delta_{ii'} \delta_{\mu\mu'} \delta_{\nu\nu'}$$

$$(\delta_{ij}=1 \text{ if } i=j \text{ and } \delta_{ij}=0 \text{ if } i \neq j)$$

(v) Orthogonality of characters:

$$\sum_R \chi^{(i)}(R)^* \chi^{(j)}(R) = h \delta_{ij}$$

(vi) Decomposition of a direct product, reduction of a representation: If

$$\Gamma = \sum_i a_i \Gamma^{(i)}$$

and the character of the operation R in the reducible representation is $\chi(R)$, then the coefficients a_i are given by

$$a_i = (l/h) \sum_R \chi^{(i)}(R)^* \chi(R).$$

(vii) Projection operators:

The projection operator

$$P^{(i)} = (l_i / h) \sum_R \chi^{(i)}(R)^* R$$

when applied to a function f , generates a sum of functions that constitute a component of a basis for the representation $\Gamma^{(i)}$; in order to generate the complete basis $P^{(i)}$ must be applied to l_i distinct functions f . The resulting functions may be made mutually orthogonal. When $l_i = 1$ the function generated is a basis for $\Gamma^{(i)}$ without ambiguity:

$$P^{(i)} f = f^{(i)}$$

(viii) Selection rules:

If $f^{(i)}$ is a member of the basis set for the irreducible representation $\Gamma^{(i)}$, $f^{(k)}$ a member of that for $\Gamma^{(k)}$, and $\hat{\Omega}^{(j)}$ an operator that is a basis for $\Gamma^{(j)}$, then the integral

$$\int d\tau f^{(i)*} \hat{\Omega}^{(j)} f^{(k)}$$

is zero unless $\Gamma^{(i)}$ occurs in the decomposition of the direct product $\Gamma^{(j)} \times \Gamma^{(k)}$

(ix) The *symmetrized* direct product is written $\Gamma^{(i)} \times^s \Gamma^{(i)}$, and its characters are given by

$$\chi^{(i)}(R) \times^s \chi^{(i)}(R) = \frac{1}{2} \chi^{(i)}(R)^2 + \frac{1}{2} \chi^{(i)}(R^2)$$

The *antisymmetrized* direct product is written $\Gamma^{(i)} \times^a \Gamma^{(i)}$ and its characters are given by

$$\chi^{(i)}(R) \times^a \chi^{(i)}(R) = \frac{1}{2} \chi^{(i)}(R)^2 - \frac{1}{2} \chi^{(i)}(R^2)$$

Worked examples

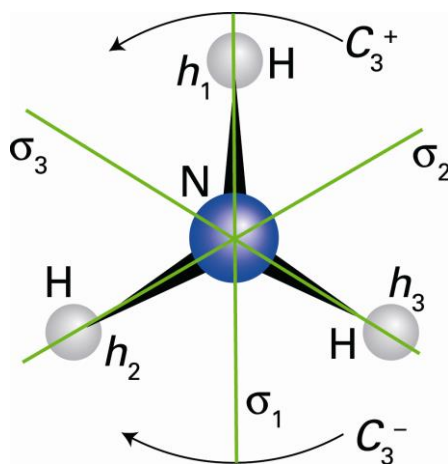
1. To show that the representation Γ based on the hydrogen 1s-orbitals in NH_3 (C_{3v}) contains A_1 and E , and to generate appropriate symmetry adapted combinations.

A table in which symmetry elements in the same class are distinguished will be employed:

C_{3v}	E	C_3^+	C_3^-	σ_1	σ_2	σ_3
A_1	1	1	1	1	1	1
A_2	1	1	1	-1	-1	-1
E	2	-1	-1	0	0	0
$D(R)$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
$x(R)$	3	0	0	1	1	1
Rh_1	h_1	h_2	h_3	h_1	h_3	h_2
Rh_2	h_2	h_3	h_1	h_3	h_2	h_1

The *representative matrices* are derived from the effect of the operation R on the basis (h_1, h_2, h_3); see the figure below. For example

$$C_3^+(h_1, h_2, h_3) = (h_2, h_3, h_1) = (h_1, h_2, h_3) \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$



According to the general formula (b)(iii) the *character* $\chi(R)$ is the sum of the diagonal elements of $\mathbf{D}(R)$. For example, $\chi(\sigma_2) = 0 + 1 + 0 = 1$. The *decomposition* of Γ follows from the formula (b)(vi):

$$\Gamma = a_1 A_1 + a_2 A_2 + a_E E$$

where

$$a_1 = \frac{1}{6} \{1 \times 3 + 2 \times 1 \times 0 + 3 \times 1 \times 1\} = 1$$

$$a_2 = \frac{1}{6} \{1 \times 3 + 2 \times 1 \times 0 + 3 \times 1 \times (-1)\} = 0$$

$$a_E = \frac{1}{6} \{2 \times 3 + 2 \times (-1) \times 0 + 3 \times 0 \times 1\} = 1$$

Therefore

$$\Gamma = A_1 + E$$

Symmetry adapted combinations are generated by the projection operator in (b)(vii). Using the last two rows of the table,

$$\begin{aligned} \phi(A_1) &= \mathcal{S}^{(A_1)} h_1 = \frac{1}{6} (1 \times h_1 + 1 \times h_2 + 1 \times h_3 + 1 \times h_1 \\ &\quad + 1 \times h_3 + 1 \times h_2) = \frac{1}{3} (h_1 + h_2 + h_3) \\ \left\{ \begin{aligned} \phi(E) &= \mathcal{S}^{(E)} h_1 = \frac{2}{6} (2 \times h_1 - 1 \times h_2 - 1 \times h_3 + 0 \times h_1 \\ &\quad + 0 \times h_3 + 0 \times h_2) = \frac{1}{3} (2h_1 - h_2 - h_3) \\ \phi'(E) &= \mathcal{S}^{(E)} h_2 = \frac{2}{6} (2 \times h_2 - 1 \times h_3 - 1 \times h_1 + 0 \times h_3 \\ &\quad + 0 \times h_2 + 0 \times h_1) = \frac{1}{3} (-h_1 + 2h_2 - h_3) \end{aligned} \right. \end{aligned}$$

$\phi(E)$ and $\phi'(E)$ provide a valid basis for the E representation, but the orthogonal combinations

$$\begin{aligned} \phi_a(E) &= (1/6)^{\frac{1}{2}} (2h_1 - h_2 - h_3) = (3/2)^{\frac{1}{2}} \phi(E) \\ \phi_b(E) &= (1/2)^{\frac{1}{2}} (h_2 - h_3) = (1/2)^{\frac{1}{2}} \{ \phi(E) + 2\phi'(E) \} \end{aligned}$$

would be a more useful basis in most applications.

2. To determine the symmetries of the states arising from the electronic configurations e^2 and $e^1 t_2^1$ for a tetrahedral complex (T_d), and to determine the group theoretical selection rules for electric dipole transitions between them.

The spatial symmetries of the required states are given by the direct products in Table 7.

$$E \times E = A_1 + [A_2] + E \qquad E \times T_2 = T_1 + T_2$$

Combination of the electron spins yields both singlet and triplet states, but for the e^2 configuration some possibilities are excluded. Since the total (spin and orbital) state must be antisymmetric under electron interchange, the antisymmetrized spatial combination $[A_2]$ must be a triplet, and the symmetrized combinations A_1 and E are singlets. For the $e^1 t_2^1$ configuration there are no exclusions. The required terms are therefore

$$\begin{aligned} e^2 &\rightarrow {}^1A_1 + {}^3A_2 + {}^1E \\ e^1 t_2^1 &\rightarrow {}^1T_1 + {}^1T_2 + {}^3T_1 + {}^3T_2 \end{aligned}$$

The selection rules are obtained from formula (b)(viii). For electric dipole transitions the operator $\Omega^{(j)}$ has the symmetry of a vector (x, y, z), which from the character table for T_d transforms as T_2 . From the table of direct products, Table 7,

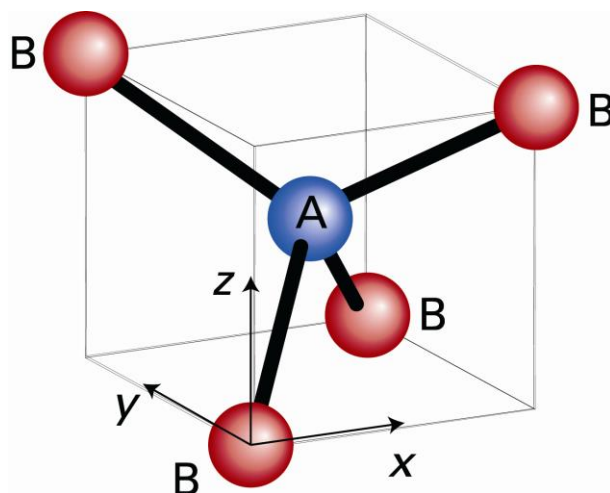
$$A_1 \times T_2 = T_2 \qquad A_2 \times T_1 = T_2 \qquad E \times T_2 = E \times T_1 = T_1 + T_2$$

Assuming the spin selection rule $\Delta S = 0$, the allowed transitions are

$$e^2 {}^1A_1 \leftrightarrow e^1 t_2^1 {}^1T_2 \qquad e^2 {}^3A_2 \leftrightarrow e^1 t_2^1 {}^3T_1 \qquad e^2 {}^1E \leftrightarrow e^1 t_2^1 {}^1T_1, {}^1T_2$$

3. To determine the symmetries of the vibrations of a tetrahedral molecule AB_4 , and to predict the appearance of its infrared and Raman spectra.

The molecule is depicted in the figure below and the character table for the point group T_d is given on page 15.



The representations spanned by the vibrational coordinates are based on the 5×3 cartesian displacements less the representations T_1 and T_2 , which are accounted for by the rotations (R_x, R_y, R_z) and the translations (x, y, z). The stretching vibrations are the subset based on the 4 bonds of the molecule. For a particular symmetry operation, only atoms (or bonds) that remain invariant can contribute to the character of the cartesian displacement representation, $\Gamma^{(all)}$ (or the stretching representation, $\Gamma^{(stretch)}$).

$$C_3: \quad \begin{array}{l} \text{Two atoms invariant, } x, y, z, \text{ interchanged} \\ \text{One bond invariant} \end{array} \quad \begin{array}{l} \chi^{(all)}(C_3) = 0 \\ \chi^{(stretch)}(C_3) = 1 \end{array}$$

$$C_2(z): \quad \begin{array}{l} \text{Central atom invariant; } x, y, \text{ sign reversed, } z \text{ invariant} \\ \text{No bonds invariant} \end{array} \quad \begin{array}{l} \chi^{(all)}(C_2) = 0 \\ \chi^{(stretch)}(C_2) = 0 \end{array}$$

$$S_4(z): \quad \begin{array}{l} \text{Central atom invariant; } x, y, \text{ interchanged, } z \text{ sign reversed} \\ \text{No bonds invariant} \end{array} \quad \begin{array}{l} \chi^{(all)}(S_4) = -1 \\ \chi^{(stretch)}(S_4) = 0 \end{array}$$

$$\sigma_d(z): \quad \begin{array}{l} \text{Three atoms invariant; } x, y, \text{ interchanged, } z \text{ invariant} \\ \text{Two bonds invariant} \end{array} \quad \begin{array}{l} \chi^{(all)}(\sigma_d) = 3 \\ \chi^{(stretch)}(\sigma_d) = 2 \end{array}$$

The characters of the representations $\Gamma^{(all)}$ and $\Gamma^{(stretch)}$ are therefore

	E	$8C_3$	$3C_2$	$6S_4$	$6\sigma_d$	
$\Gamma^{(all)}$	15	0	-1	-1	3	$= A_1 + E + T_1 + 3T_2$
$\Gamma^{(stretch)}$	4	1	0	0	2	$= A_1 + T_2$

$\Gamma^{(all)}$ and $\Gamma^{(stretch)}$ have been decomposed with the help of formula (b)(vi) (compare Example 1). Allowing for the rotations and translations contained in $\Gamma^{(all)}$ there are therefore four fundamental vibrations, conventionally labelled $\nu_1(A_1)$, $\nu_2(E)$, $\nu_3(T_2)$, and $\nu_4(T_2)$. ν_1 and ν_2 are stretching and bending vibrations respectively, ν_3 and ν_4 involve both stretching and bending motions.

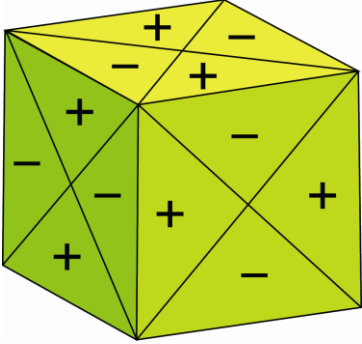
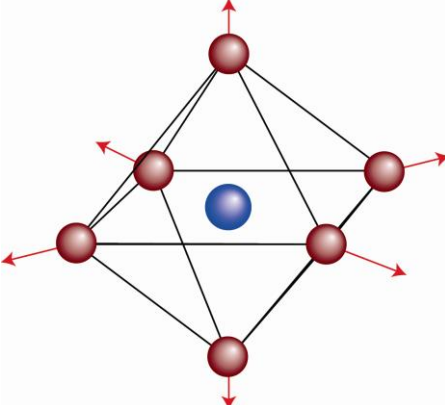
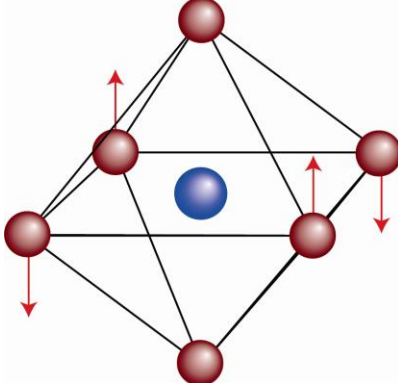
The selection rule (b)(viii) gives the spectroscopic properties of the vibrations. Infrared activity is induced by the dipole moment (a vector with symmetry T_2 , according to the character table for T_d) as the operator $\hat{\Omega}^{(j)}$. In the case of the Raman effect, $\hat{\Omega}^{(j)}$ is the component of the polarizability tensor ($A_1 + E + T_2$). $f^{(i)}$ is the ground vibrational state (A_1), and $f^{(k)}$ is the excited state (with the same symmetry as the vibration in the case of the fundamental; as the direct product of the appropriate representations in the case of an overtone or a combination band). $\nu_1(A_1)$ and $\nu_2(E)$ are therefore Raman active and $\nu_3(T_2)$ and $\nu_4(T_2)$ are infrared and Raman active. The following overtone and combination bands are allowed in the infrared spectrum:

$$\nu_1 + \nu_3, \nu_1 + \nu_4, \nu_2 + \nu_3, \nu_2 + \nu_4, 2\nu_3, \nu_3 + \nu_4, 2\nu_4$$

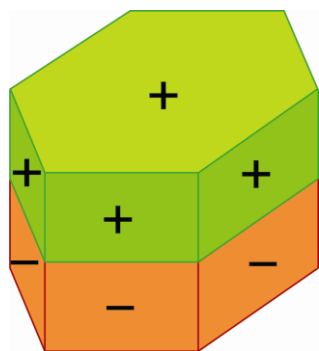
Examples of bases for some representations

The customary bases—polar vector (e.g. translation x), axial vector (e.g. rotation R_x), and tensor (e.g. xy)—are given in the character tables.

It may be of some assistance to consider other types of bases and a few examples are given here.

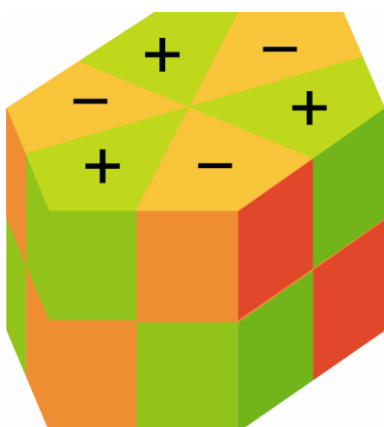
	Base	Irreducible Representation
1		A_2 in T_d
2	$x(1)y(2) - x(2)y(1)$	A_2 in C_{4v}
3	The normal vibration of an octahedral molecule represented by 	A_{1g} in O_h
4	The three equivalent normal vibrations of an octahedral molecule, one of which is represented by 	T_{2u} in O_h

5 The π -orbital of the benzene molecule represented by



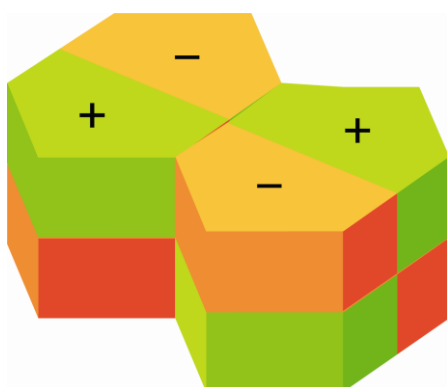
A_{2u} in D_{6h}

6 The π -orbital of the benzene molecule represented by



B_{2g} in D_{6h}

7 The π -orbital of the naphthalene molecule represented by



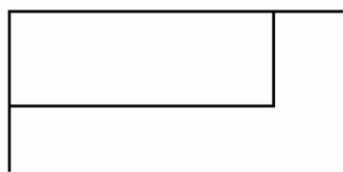
A_u in D_{2h}

Illustrative Examples of Point Groups
I Shapes

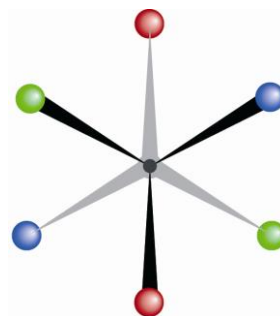
	$n = 2$	3	4	5	6	∞
C_n						
D_n						
C_{nv} (pyramid)						
C_{nh}						
D_{nh} (plane or bipyramid)						
D_{nd}						
S_{2n}						

The character tables for (a), C_n , are on page 4; for (b), D_n , on page 6; for (c), C_{nv} , on page 7; for (d), C_{nh} , on page 8; for (e), D_{nh} , on page 10; for (f), D_{nd} , on page 12; and for (g), S_{2n} , on page 14.

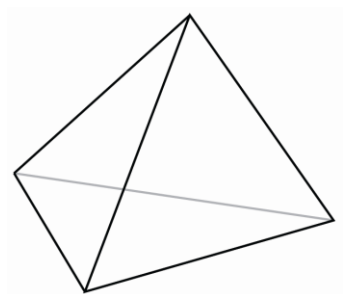
C_s



C_i

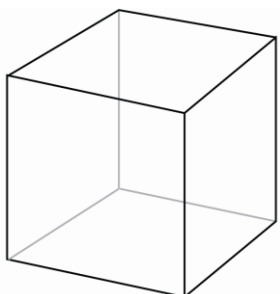


T_d



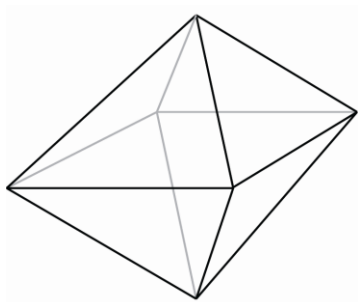
tetrahedron

O_h



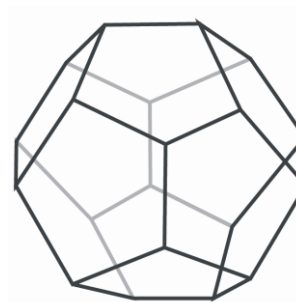
cube

O_h



octahedron

I_h



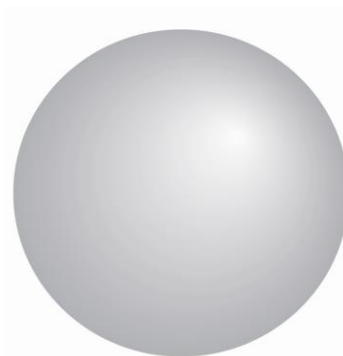
dodecahedron

I_h



icosahedron

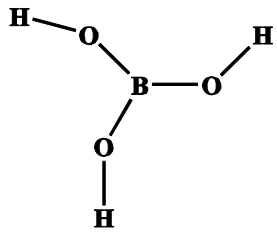
R_3



sphere

The character table for C_s is on page 3, for C_i on page 3, for T_d on page 15, for O_h on page 16, for I_h on page 17, and for R_3 on page 19.

II Molecules

Point group	Example	Page number for character table
C_1	CHFCIBr	3
C_s	BFCIBr (planar), quinoline	3
C_i	meso-tartaric acid	3
C_2	H_2O_2 , S_2Cl_2 (skew)	4
C_{2v}	H_2O , HCHO, C_6H_5Cl	7
C_{3v}	NH_3 (pyramidal), $POCl_3$	7
C_{4v}	SF_5Cl , $XeOF_4$	7
C_{2h}	<i>trans</i> -dichloroethylene	8
C_{3h}	 <p>(in planar configuration)</p>	8
D_{2h}	<i>trans</i> - PtX_2Y_2 , C_2H_4	10
D_{3h}	BF_3 (planar), PCl_5 (trigonal bipyramid), 1:3:5-trichlorobenzene	10
D_{4h}	$AuCl_4^-$ (square plane)	10
D_{5h}	ruthenocene (pentagonal prism), IF_7 (pentagonal bipyramid)	11
D_{6h}	benzene	11
D_{2d}	$CH_2=C=CH_2$	12
D_{4d}	S_8 (puckered ring)	12
D_{5d}	ferrocene (pentagonal antiprism)	13
S_4	tetraphenylmethane	14
T_d	CCl_4	15
O_h	SF_6 , FeF_6^{3-}	16
I_h	$B_{12}H_{12}^{2-}$	17
$C_{\infty v}$	HCN, COS	18
$D_{\infty h}$	CO_2 , C_2H_2	18
R_3	any atom (sphere)	19