THÉVENIN'S THEOREM

APPENDIX D

In Chapter 1 we stated (but did not "prove") Thévenin's Theorem, namely that any two-terminal network whose internal circuitry consists solely of resistors, batteries, and current sources, interconnected in any manner whatsoever, is equivalent (and indistinguishable) from the two-terminal network consisting of a single battery V_{TH} in series with a single resistor R_{TH};[1] see Figure D.1. We did not prove it, because, in the spirit of this book, we don't *prove* anything; we show you how to design circuits, instead. We make an exception here, because it's nice to see *something* proved, right?

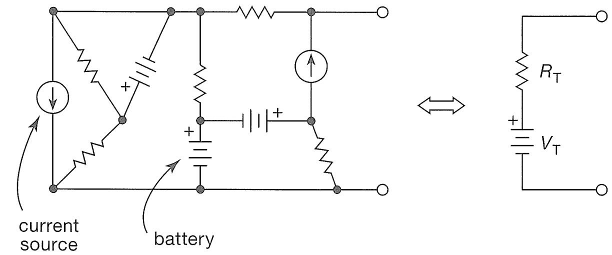


Figure D.1. Thévenin's Theorem: a single resistor in series with a single battery can mimic any mess of a two-terminal network made from resistors, batteries, and current sources.

D.1 The proof

For linear circuit elements (here resistors), the "nodal equations" (Kirchhoff's voltage law, KVL, and Kirchhoff's current law, KCL) are a set of linear equations. So we can find any circuit quantity (a voltage or a current), which depends on all the "independent sources" (batteries, current sources), by turning on each source in turn, and adding the partial contributions. (This is exactly analogous to using superposition to find, say, the electric field from a set of charges.) This technique is often useful in circuit analysis.

Here we wish to mimic the V versus I of the actual circuit with the (simpler) Thévenin equivalent of a single battery in series with a single resistor. Imagine we determine that V versus I function by applying an external current I_{ext} that flows through the two-terminal circuit, and observing the resultant V across those same two terminals. V depends on I_{ext} *and* on all the internal batteries (V_{int}) and current sources (I_{int}).

1. Set all $V_{\mathrm{int}}=0$ and all $I_{\mathrm{int}}=0$; that is, replace all internal batteries with short circuits and all current sources with open circuits. Now, with a given applied I_{ext}, observe V_1.
2. Define $R_{\mathrm{T}}=V_1/I_{\mathrm{ext}}$. (They must be proportional, by linearity.)
3. Now set $I_{\mathrm{ext}}=0$, and turn on the internal batteries and current sources. Observe V_2, which we will call V_{T}.
4. Finally, by superposition it must be the case that

$$V(\text{actual})=V_1+V_2=I_{\mathrm{ext}}R_{\mathrm{T}}+V_{\mathrm{T}}.$$

This is true for all I_{ext}, and is exactly what you get with the Thévenin equivalent circuit, when connected to *any* load (which need not be linear); see Figure D.2.

To summarize: (a) you determine R_{T} and V_{T} by first finding the open-circuit voltage, which equals V_{T}; then (b) you find the short-circuit current, I_{SC}, which equals the ratio of V_{T} to R_{T}. In other words, $V_{\mathrm{T}}=V_{\mathrm{OC}}$ and $R_{\mathrm{T}}=V_{\mathrm{OC}}/I_{\mathrm{SC}}$. You do this by analysis, if you know the "black-box" circuit; or by measurement, if you don't.

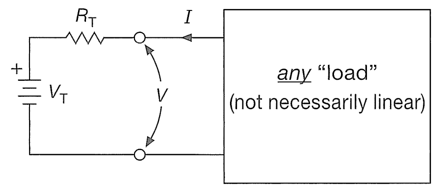


Figure D.2. The Thévenin equivalent circuit behaves exactly like the original network, regardless of the nature of the load.

D.1.1 Two examples – voltage dividers

Figures D.3 and D.4 show two simple examples, variations on the resistive divider. Interestingly, their Thévenin equivalent circuits are different, even though the resistor values and the open-circuit voltages are the same.

[1] A related theorem is Norton's, where the equivalent circuit consists of a resistor R_{N} in parallel with a current source I_{N}.

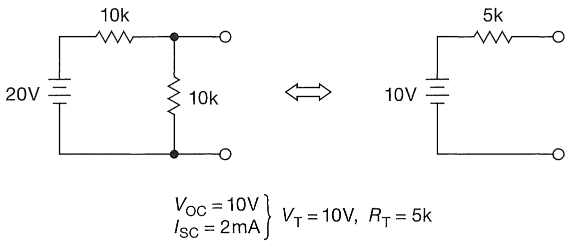


Figure D.3. Thévenin equivalent of a simple resistive divider. Note that R_T is the parallel resistance of the divider (as if the voltage source were replaced with a short circuit).

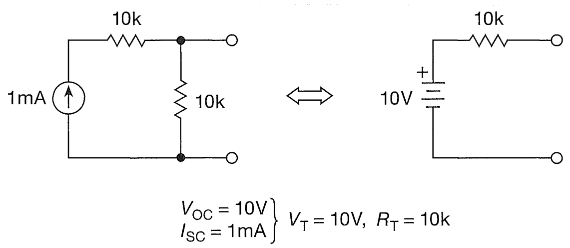


Figure D.4. Note that the Thévenin equivalent resistance is here *not* equal to the parallel resistance of the divider components. Instead it equals the value of the resistor across the output alone (as if the current source were replaced with an open circuit).

D.2 Norton's theorem

You can replace a Thévenin circuit with a Norton circuit, which consists of a current source I_N in parallel with a resistor R_N (Figure D.5). It is easy to show that $I_N = I_{SC}$ and $R_N = R_T$ ($= V_{OC}/I_{SC}$). So, for the two examples above, the Norton equivalents are as shown in Figure D.6.

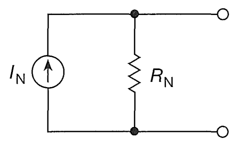


Figure D.5. Norton equivalent circuit: a current source in parallel with a resistor.

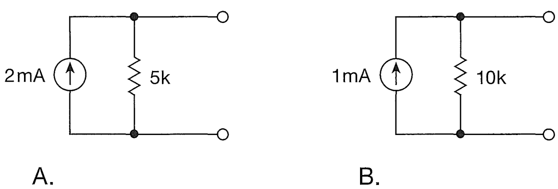


Figure D.6. Norton equivalents of the circuits of Figure D.3 (A) and Figure D.4 (B).

D.3 Another example

Figure D.7 shows a complicated-looking circuit, for which it is pretty easy to see that V_{OC}=25 V (the bottom of the 10k resistor sits at +10 V, and 1.5 mA flows into the top) and that I_{SC}=2.5 mA (10 V across the 10 k, plus the two current sources). From that you get the equivalent circuits shown.

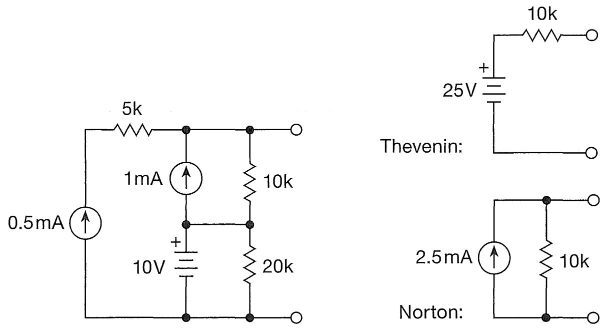
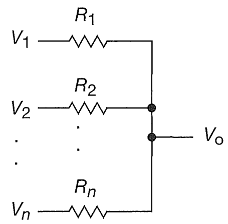


Figure D.7. Thévenin and Norton equivalents of a complicated-looking circuit.



$$V_o = \frac{\Sigma V_i/R_i}{\Sigma 1/R_i} = \frac{\Sigma V_i G_i}{\Sigma G_i}$$

$(G_i \equiv 1/R_i)$

Figure D.8. Millman's theorem for parallel circuits.

D.4 Millman's theorem

A related – and useful – tool is *Millman's Theorem* (also known as the parallel generator theorem), which is helpful when dealing with circuits with several parallel branches. It's shown in Figure D.8, where a set of input voltages V_i are combined via resistors R_i, producing an output voltage V_o. The latter is just $V_o = (\sum V_i G_i)/\sum G_i$, where the G_i are the conductances $G_i \equiv 1/R_i$. The input voltages V_i can of course include ground, forming a voltage divider. Millman's theorem, which comes from the more general class of network theorems, can be generalized to include input *currents* I_k, whose sum is added to the numerator (but whose series resistances, if any, do not appear in the denominator).