LC BUTTERWORTH FILTERS

APPENDIX E

Active filters (see Chapter 6) are convenient at low frequencies, but they are impractical at radio frequencies because of the slew-rate and bandwidth requirements they impose on the operational amplifiers. At frequencies of 100 kHz and above (and often at lower frequencies), the best approach is to design a passive filter with inductors and capacitors. (At UHF and microwave frequencies these "lumped-component" filters are replaced by stripline and cavity filters.)

As with active filters, there are many methods and filter characteristics possible with *LC* filters. For example, you can design the classic Butterworth, Chebyshev, and Bessel filters, each in lowpass, bandpass, highpass, and band-reject varieties. It turns out that the Butterworth filter is particularly easy to design, and we can present in just a page or two all the essential design information for lowpass and highpass Butterworth *LC* filters, and even a few examples.

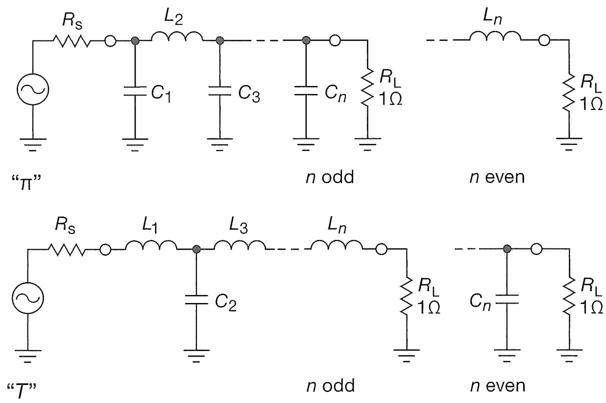

Figure E.1. π and T configurations. See Table E.1 and text.

E.1 Lowpass filter

Table E.1 gives the values of normalized inductances and capacitances for low-pass filters of various orders, from which actual circuit values are obtained by the frequency and impedance scaling rules

$$L_n(\text{actual}) = \frac{R_L L_n(\text{table})}{\omega},$$
$$C_n(\text{actual}) = \frac{C_n(\text{table})}{\omega R_L},$$

where R_L is the load impedance and ω is the angular frequency ($\omega{=}2\pi f$).

Table E.1 gives normalized values for two-pole through eight-pole lowpass filters for the two most common cases, namely (a) equal source and load impedances and (b) either source or load impedance much larger than the other. To use the table, first decide how many poles you need, based on the Butterworth response (graphs are plotted in Figure 6.30). Then use the preceding equations to determine the filter configuration (T or π; see Figure E.1) and component values. For equal source and load impedances, either configuration is OK; the π configuration may be preferable because it requires fewer inductors. For a load impedance much higher (lower) than the source impedance, use the T (π) configuration.

E.2 Highpass filter

To design a highpass filter, follow the same procedure to determine which filter configuration to use and how many poles are necessary. Then do the universal lowpass to highpass transformation shown in Figure E.2, which consists simply of replacing inductors by capacitors, and vice versa. The actual component values are determined from the normalized values in Table E.1 by the following frequency and impedance scaling rules:

$$L_n(\text{actual}) = \frac{R_L}{\omega C_n(\text{table})},$$
$$C_n(\text{actual}) = \frac{1}{R_L \omega L_n(\text{table})}.$$

E.3 Filter examples

Here are a few examples showing how to design both lowpass and highpass filters.

Example I. Design a five-pole lowpass filter for source

Table E.1 Butterworth Lowpass Filters[a]

	$\pi \longrightarrow R_S$	C_1	L_2	C_3	L_4	C_5	L_6	C_7	L_8
	$T \longrightarrow 1/R_S$	L_1	C_2	L_3	C_4	L_5	C_6	L_7	C_8
$n = 2$	1	1.4142	1.4142						
	∞	1.4142	0.7071						
$n = 3$	1	1.0000	2.0000	1.0000					
	∞	1.5000	1.3333	0.5000					
$n = 4$	1	0.7654	1.8478	1.8478	0.7654				
	∞	1.5307	1.5772	1.0824	0.3827				
$n = 5$	1	0.6180	1.6180	2.0000	1.6180	0.6180			
	∞	1.5451	1.6944	1.3820	0.8944	0.3090			
$n = 6$	1	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176		
	∞	1.5529	1.7593	1.5529	1.2016	0.7579	0.2588		
$n = 7$	1	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	
	∞	1.5576	1.7988	1.6588	1.3972	1.0550	0.6560	0.2225	
$n = 8$	1	0.3902	1.1111	1.6629	1.9616	1.9616	1.6629	1.1111	0.3902
	∞	1.5607	1.8246	1.7287	1.5283	1.2588	0.9371	0.5776	0.1951

Notes: (a) Values of L_n, C_n for 1Ω load resistance, and cutoff frequency (–3dB) of 1 rad/s. See text for scaling rules.

and load impedances of 75 Ω, with a cutoff frequency (−3 dB) of 1 MHz.

We use the π configuration to minimize the number of required inductors. The scaling rules give us

$$C_1 = C_5 = \frac{0.618}{2\pi \times 10^6 \times 75} = 1310\,\text{pF},$$

$$L_2 = L_4 = \frac{75 \times 1.618}{2\pi \times 10^6} = 19.3\,\mu\text{H},$$

$$C_3 = \frac{2}{2\pi \times 10^6 \times 75} = 4240\,\text{pF}.$$

The complete filter is shown in Figure E.3. Note that all filters with equal source and load impedances will be symmetrical.

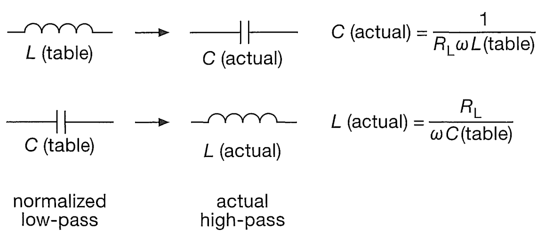

Figure E.2. Lowpass to highpass transformation.

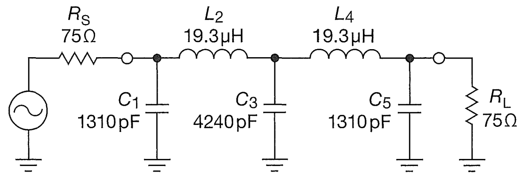

Figure E.3. Circuit for Example I. Five-pole 1 MHz lowpass with equal source and load impedances.

Example II. Design a three-pole lowpass filter for a source impedance of 50 Ω and a load impedance of 10k, with a cutoff frequency of 100 kHz.

We use the T configuration, because $R_S \ll R_L$. For R_L=10k, the scaling rules give

$$L_1 = \frac{10^4 \times 1.5}{2\pi \times 10^5} = 23.9\,\text{mH},$$

$$C_2 = \frac{1.3333}{2\pi \times 10^5 \times 10^4} = 212\,\text{pF},$$

$$L_3 = \frac{10^4 \times 0.5}{2\pi \times 10^5} = 7.96\,\text{mH}.$$

The complete filter is shown in Figure E.4.

Example III. Design a four-pole lowpass filter for a zero-impedance source (voltage source) and a 75 Ω load, with a cutoff frequency of 10MHz.

We use the T configuration, as in the previous example,

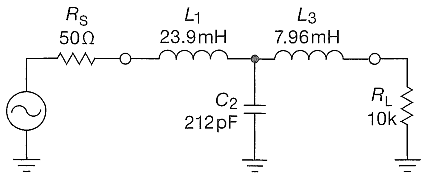

Figure E.4. Circuit for Example II. Three-pole 100 kHz lowpass with 50 Ω source and 10k load.

because $R_S \ll R_L$. The scaling rules give

$$L_1 = \frac{75 \times 1.5307}{2\pi \times 10^7} = 1.83\,\mu\text{H},$$

$$C_2 = \frac{1.5772}{2\pi \times 10^7 \times 75} = 335\,\text{pF},$$

$$L_3 = \frac{75 \times 1.0824}{2\pi \times 10^7} = 1.29\,\mu\text{H},$$

$$C_4 = \frac{0.3827}{2\pi \times 10^7 \times 75} = 81.2\,\text{pF}.$$

The complete filter is shown in Figure E.5.

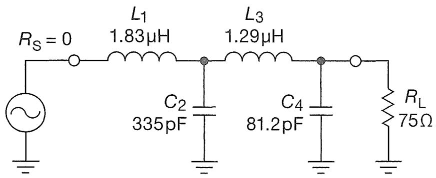

Figure E.5. Circuit for Example III. Four-pole 10 MHz lowpass with voltage source and 75 Ω load.

Example IV. Design a two-pole lowpass filter for current-source drive and 1k load impedance, with a cutoff frequency of 10 kHz.

We use the π configuration because $R_S \gg R_L$. The scaling rules give

$$C_1 = \frac{1.4142}{2\pi \times 10^4 \times 10^3} = 0.0225\,\mu\text{F},$$

$$L_2 = \frac{10^3 \times 0.7071}{2\pi \times 10^4} = 11.3\,\text{mH}.$$

The complete filter is shown in Figure E.6.

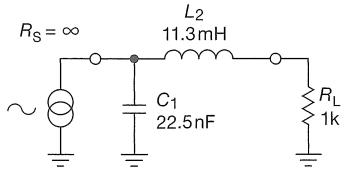

Figure E.6. Circuit for Example IV. Two-pole 10 MHz lowpass with current source drive and 1k load.

Example V. Design a three-pole highpass filter for 52 Ω source and load impedances, with a cutoff frequency of 6 MHz.

We begin with the T configuration, then transform inductors to capacitors, and vice versa, giving

$$C_1 = C_3 = \frac{1}{52 \times 2\pi \times 6 \times 10^6 \times 1.0} = 510\,\text{pF},$$

$$L_2 = \frac{52}{2\pi \times 6 \times 10^6 \times 2.0} = 0.690\,\mu\text{H}.$$

The complete filter is shown in Figure E.7.

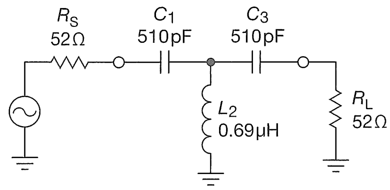

Figure E.7. Circuit for Example V. Three-pole 6 MHz highpass with equal source and load impedances.

We would like to emphasize that the field of passive filter design is rich and varied and that this simple table of Butterworth filters doesn't even begin to scratch the surface.